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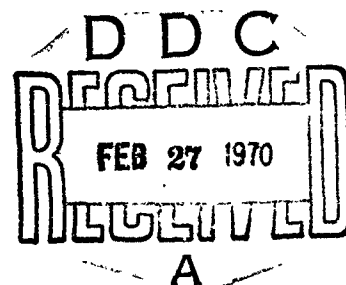
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MAGNETIC SHIELDING BY THIN WALLED  
SUPERCONDUCTING SHELLS

By  
Peter Scharnhorst



7 JANUARY 1970

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UNITED STATES NAVAL ORDNANCE LABORATORY, WHITE OAK, MARYLAND

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MAGNETIC SHIELDING BY THIN WALLED SUPERCONDUCTING SHELLS

by:

Peter Scharnhorst

ABSTRACT: A general discussion of the magnetic shielding strength of thin walled ( $d \ll \lambda$ ) superconducting shells of arbitrary shape in arbitrary magnetic fields is given. It is shown that the ratio of the self-inductance of the shell and the kinetic mass of the induced supercurrents,  $(L/m)$ , represents a sufficiently accurate estimate of the shielding strength of the shell. The ratio  $(L/m)$  turns out to be an approximately known function of only one essential length parameter which can be determined at a glance for any magnetic screening problem encountered in practice.

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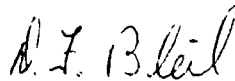
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**MAGNETIC SHIELDING BY THIN WALLED SUPERCONDUCTING SHELLS**

The research reported herein was carried out in the Electronics and Electromagnetics Division of the Physics Research Department as a part of the Independent Research Program (Task No. MAT 03L 000/ZR 011 01 01), Superconductivity Research. This report is for information only.

GEORGE G. BALL

A handwritten signature in cursive script, appearing to read "D. F. Bleil".

D. F. BLEIL  
By direction

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2. see, e.g., "Superconductivity of Metals and Alloys", P. G. DeGennes, W. A. Benjamin, Inc. (1966), p. 183 where it is shown that, for the case of uniform current flow,  $1 \geq \theta^2 \geq 2/3$ .
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4. see, e.g., D. H. Douglass, Jr., Phys. Rev., Vol 132, No 2, 513 (1963).

# INTRODUCTION

It is well known that the internal magnetic field of a bulk superconductor in an external field,  $H_0$ , decreases exponentially toward the interior of the specimen. The same field penetration law predicts that the field inside a thick walled superconducting shell,  $H_i$ , is likewise exponentially attenuated;  $H_i/H_0 \approx e^{-d/\lambda} \ll 1$ . If, on the other hand, this law is applied to a thin walled shell with  $d \ll \lambda$ , it predicts  $H_i \approx H_0$ , i.e. it predicts that field attenuation is extremely weak.

We wish to point out that in the case of a thin walled superconducting shell with  $d \ll \lambda$ , contrary to the conclusion just reached on the basis of the bulk penetration law, the internal field is strongly attenuated. It will be shown that an attenuation factor of the order of  $10^4 (= H_0/H_i)$  is easily attainable with a thin walled shell of mean diameter  $\geq 1$  cm, even if the shell consists only of a thin film of order 100 Å thick; even if the thickness of the shell is much smaller than the penetration depth,  $\lambda$ . Further, the attenuation factor for a double shell is found to be a rapidly increasing function of the ratio of the mean shell radii. It will be shown by way of example that for the special configuration of concentric spherical shells, the attenuation factor approaches  $(R'/R) \times$  the square of the single shell attenuation factor as  $[1 - (R'/R)^3] \rightarrow 1$ , where  $R'$  and  $R$  are the mean shell radii of the inner and outer shells respectively. In general, the  $n$ -shell factor is

$$9 \pi_{i=1}^n \left( \frac{R_i}{R_1} \right) \alpha^n$$

as the function

$$\pi_{i=1}^n \left[ 1 - \left( \frac{R_{i+1}}{R_i} \right)^3 \right] \rightarrow 1$$

where  $\left( \frac{R_{i+1}}{R_i} \right)$  is the ratio of concentric mean shell radii ( $R_{i+1} < R_i$ ).



A rigorous derivation of the attenuation law for arbitrarily shaped shells is too ambitious. Part of the discussion is therefore confined to the perfectly spherical shell. Certain physical ideas presented at the end will, however, suggest that the results obtained for the spherical shell are indeed quite general. We will argue that for shells of arbitrary shapes in arbitrary fields, the quantity  $\alpha' = [1 + (L/m)]^{-1}$  where  $L$  is the self-inductance of the shell and  $m$  is the kinetic mass of the induced supercurrents, yields a sufficiently accurate estimate of the attenuation. We show that the magnitude of  $(L/m)$  is an approximately known function of only one essential length parameter which can be determined at a glance for any magnetic screening problem encountered in practice.

### DISCUSSION OF MAGNETIC SHIELDING

The response of the shell system to an arbitrarily applied field is governed by the following equations:<sup>1</sup>

$$\nabla^2 \phi = K[\phi^2 + Q^2 - 1] \phi$$

$$\nabla \times \nabla \times \vec{Q} = -\phi^2 \vec{Q}$$

where the order parameter  $f = \phi(x, y, z) e^{-i\psi(x, y, z)}$ ,  $\vec{Q} \equiv \nabla\psi/K - \vec{A}$ ,  $\vec{A}$  is the vector potential,  $K = \lambda/\xi$ ,  $\lambda(T)$  = penetration depth, and  $\xi(T)$  = coherence length. We consider the pure Meissner state;  $\psi = 0$ :

$$\nabla^2 \phi = K[\phi^2 + A^2 - 1] \phi \quad (1)$$

$$\nabla \times \nabla \times \vec{A} = -\phi^2 \vec{A} \quad (2)$$

In order to obtain a solution of these equations, we concentrate on the second one and assume as a first approximation that  $\phi \approx$  constant  $\approx 1$ . A discussion of this approximation may be found in the literature.<sup>2</sup> It turns out that the assumption leads to results which are in agreement with experiment up to applied fields of the order of 0.5 gauss at temperatures  $T < T_{c/2}$ ,<sup>3</sup> provided, as a rule, that the curvature of the shell is fairly uniform. If the radius of curvature of part of the shell is too small (of order  $\lambda$ ), as is the case, for instance, for an extremely oblate spheroidal shell, the critical field may be much less than 0.5 gauss. This will be the case if the large dimension of such a shell is placed transverse to an external field.

With  $\emptyset = \text{constant}$ , the solution of equation (2) is straightforward and we merely state the results. The internal magnetic field of a spherical double shell in an applied field,  $H_0$ , is:

$$\left[ \frac{H_i}{H_0} \right] = \frac{9}{(\emptyset a)^2 R_2 R_4 R_3} \left\{ (\emptyset a)^4 R_2^2 R_4^2 \left( R_3^{-3} [n_2(R_2) j_0(R_1) + j_0(R_3) n_2(R_4)] \right. \right. \\ \left. \left. - R_2^{-3} [n_0(R_2) j_0(R_1) + j_0(R_3) n_2(R_4)] \right) \right\}^{-1} \quad (3)$$

where  $a = \lambda^{-1}$ ,  $R_1$  to  $R_4$  are concentric inner and outer radii of the shells:  $R_1 > R_2 > R_3 > R_4$ .  $[n_2 j_0]$ , e.g. stands for  $[n_2 j_0 - j_2 n_0]$  and, e.g.  $j(R) \equiv j[i\emptyset a R]$ . The  $n$ 's and the  $j$ 's are spherical Bessel functions.

For  $\emptyset a R_i \gg 1$ , all  $i$ 's; and  $\emptyset a d_j \ll 1$ ,  $j = 1, 2$ .

$$\left[ \frac{H_i}{H_0} \right] = \frac{9 R_1 R_3}{R_2 R_4} \left\{ (\emptyset a)^4 R_2 R_4 d_1 d_2 [1 - (R_3/R_2)^3] + 3(\emptyset a)^2 \right. \\ \left. [R_4 d_2 + R_2 d_1] + 3(\emptyset a)^2 R_2 d_1 \left( \frac{R_3}{R_2} \right)^3 \left[ \frac{R_4}{R_3} - 1 \right] \right\}^{-1}$$

$d_1$ ;  $d_2$  = thickness of the outer and inner shell respectively. This is the desired result.

The attenuation factor is dominated by the function in the curly brackets. For a single shell of thickness  $d$  this reduces to  $3(\emptyset a)^2 R d$ , which is of order  $10^4$  for  $\emptyset a R = 10^5$  and  $\emptyset a d = 10^{-1}$ . The latter values of the parameters are easily attained in practice; e.g.  $\lambda = 10^{-5}$  cm,  $d = 10^{-6}$  cm,  $R = 1$  cm,  $\emptyset \approx 1$ . Further, as  $(R_3/R_2)$  becomes  $< 1$ , the attenuation approaches

$$9[(\emptyset a)^2 R d_1 \times (\emptyset a)^2 R' d_2]^{-1} \text{ as } [1 - (R'/R)^3] \rightarrow 1$$

where  $R'$  and  $R$  are the mean radii of the inner and outer shell respectively. Since  $R'$  and  $R$  need not be very different to make  $[1 - (R'/R)^3] \approx 1$ , the double shell attenuation factor may easily be of order  $10^8$ , even if each shell is extremely thin, i.e.  $d_1/\lambda$ ,  $d_2/\lambda \ll 1$ .

Next, a word about the general validity of this result for arbitrarily shaped shells. First, two more particular examples: For the infinite cylindrical shell in an axial magnetic field,  $H_0$ , one finds:<sup>4</sup>

$$\alpha = \frac{H_i}{H_0} = [1 + (\delta a)^2 R d]^{-1}.$$

It is also easy to guess at the attenuation for a cylindrical shell in a transverse field by comparing with the latter formula and the formula for the single spherical shell which follows from equation (3)

$$\alpha = [1 + \frac{1}{3}(\delta a)^2 R d]^{-1};$$

it should be

$$\alpha = [1 + \frac{1}{2}(\delta a)^2 R d]^{-1}$$

where the factors, 1, 1/2, 1/3 are the respective demagnetization factors.

In order to arrive at a general statement and at some physical insight, we consider the following expression for a superconducting shell of arbitrary shape, placed in an arbitrary magnetic field:

$$\alpha' = \frac{\iint_C \vec{J}(r) \cdot \vec{A}(r) dl_c(r) da(r)}{\iint_C \vec{J}(r) \cdot \vec{A}_0(r) dl_c(r) da(r)}$$

$J(r)$  is the current density,  $A_0$  is the applied vector potential, the integral  $\int_C dl_c(r)$  is taken along the current flow pattern in the wall of the shell and the integral  $\int da(r)$  is taken perpendicular to  $dl_c(r)$  over the current carrying cross section of the shell.

The two integrals in  $\alpha'$  are proportional to familiar expressions for the magnetic field energy. In this context, however, we view each integral as a weighted average of the flux linking the superconducting shell, the weight function being the current density

$J(x,y,z)$ . Both numerator and denominator in  $\alpha'$  are similarly affected by the weight  $J(x,y,z)$ . Further the spatial dependence of  $J(x,y,z)$  is similar to the spatial dependence of the vector potentials.  $J(x,y,z)$  is, in fact, proportional to  $A(x,y,z)$  (see equation ,4) below). Hence one expects the ratio  $\alpha'$  to be relatively insensitive to  $J(x,y,z)$ . The major variation of  $\alpha'$  will come from the magnitude of the fluxes themselves.  $\alpha'$  is therefore a measure of the average magnetic field attenuation.

$\alpha'$  may be transformed into a more suggestive form. Since

$$A(r) = A_O(r) + A_R(r)$$

where  $A_R(r)$  is the response-vector potential. one may write:

$$\begin{aligned} \iint_C \vec{J}(r) \cdot \vec{A}(r) dl_C(r) da(r) &= \iint_C \vec{J}(r) \cdot \vec{A}_O(r) dl_C(r) da(r) \\ &+ \iint_C \vec{J}(r) \cdot \vec{A}_R(r) dl_C(r) da(r) \end{aligned}$$

and hence:

$$\alpha' = \left\{ 1 + \frac{\iint_C \iint_C da(r) da(r') \frac{|\vec{J}(r)| |\vec{J}(r')|}{|\vec{r}-\vec{r}'|} \vec{dl}_C(r) \cdot \vec{dl}_C(r')}{\frac{4\pi\lambda^2}{c\theta^2} \int_{\text{shell}} |\vec{J}(r)|^2 d\tau(r)} \right\}^{-1}$$

where we have used:  $\vec{A}_R(r) = \frac{1}{c} \iint_C \frac{|\vec{J}(r)| \vec{dl}_C(r) da(r)}{|\vec{r}-\vec{r}'|}$

and  $\vec{A}(r) = - \frac{4\pi\lambda^2}{c\theta^2} \vec{J}(r)$  (4)

Since  $\iint_C \iint_C da(r) da(r') \frac{|\vec{J}(r)| |\vec{J}(r')|}{|\vec{r}-\vec{r}'|} \vec{dl}_C(r) \cdot \vec{dl}_C(r')$   
 $= LI^2$  and  $\frac{4\pi\lambda^2}{c\theta^2} \int_{\text{shell}} |\vec{J}(r)|^2 d\tau(r) = mI^2$

where  $I$  = the total induced current in the shell,  $L$  = the self inductance of the shell and  $m$  = the kinetic mass of the supercurrent  $I$ :

$$\alpha' = \frac{1}{[1+(L/m)]}.$$

We therefore have the very general statement: The magnetic field attenuation of a thin walled superconducting shell ( $d/\lambda \ll 1$ ) is large if the ratio of the electromagnetic mass ( $\equiv L$ ) to the kinetic mass  $m$  of the induced supercurrent is large.

The specific examples given above yield, after some lengthy calculations of  $L$  and  $m$ , that the ratio  $(L/m)$  is, in fact, of order  $(\bar{\partial} R)^2 R d$ . These explicit calculations, as well as the dependence of  $\alpha'$  on the simple macroscopic quantities  $L$  and  $m$ , suggest that  $\alpha'$  is indeed small, or equivalently that  $(L/m)$  is large, independently of the exact form of  $J(x,y,z)$ . This is, in fact, as expected from the original definition of  $\alpha'$ .

On the other hand, the overall macroscopic dimension of the induced current distribution, which in turn determines the self-inductance of the shell, is by no means unimportant. This overall dimension of the current distribution is, in fact, the essential variable in  $(L/m)$ .

Consider again the shell in a uniform applied field. Here the dimension of the current distribution is clearly defined by the dimensions, or the size of the shell itself. The response is therefore governed by  $\bar{R}$  in  $(\bar{\partial} R)^2 R d$  where  $\bar{R}$  is a measure of the overall size of the shell. One notes in this connection, for the special examples cited above, that  $\alpha$  becomes of order one as  $(\bar{\partial} R) R \rightarrow 1$ ; field penetration becomes strong for  $d \ll \lambda$  only in the limit of very small shell dimensions. In general, field penetration is strong, or  $\alpha' \approx 1$  when the magnetic inductance  $L$  is comparable to the kinetic inductance  $m$  of the supercurrent.

If now the applied field is nonuniform over the volume of the shell, then the induced current distribution will be determined to some extent by the distribution of the external currents. As the dimensions of the external current distribution became small with respect to the size of the shell, the shell will begin to mirror this distribution. If the external current distribution is placed in close proximity with the shell, the induced current distribution takes on the overall dimensions of the external distribution.

Suppose that the overall size parameter of this distribution is "a". The self-inductance of the shell is now determined by a current distribution "of size a", and the attenuation  $[(\theta a)^2 R_2 d]^{-1}$  is replaced by an expression of the form  $[(\theta a)^2 f(a) d]^{-1}$ . "a" is the new essential size parameter.  $f(a)$  is at least of order "a" and one notes that magnetic field attenuation will remain strong, regardless of the nonuniformity of the applied field as long as  $[(\theta a)^2 a d]$  remains large; as long as the dimensions of the external current distribution remain large compared to both d and  $\lambda$ . This is always the case in practice.

We can illustrate these ideas by considering the response of a spherical shell to a single current loop of unit strength. The vector potential inside the shell is easily shown to be:

$$\vec{A} = -i \frac{4\pi}{c} \frac{1}{(\theta R_2)^3} \sum_{n=1}^{\infty} \frac{(2n+1)}{2} \frac{(n-1)!}{(n+1)!} \left(\frac{R_1}{R_2}\right)^{n-1} \left(\frac{R_1}{a}\right)^n P_n^1(\cos\theta_0)$$

$$P_n^1(\cos\theta) \left[ \frac{n}{n+1} (i\theta a R_2)^n \frac{n-1}{n-1} (i\theta a R_1)^{n-1} \right]^{-1}$$

where  $(a, \theta_0)$  are the position coordinates of the loop. Using the fact that the field at the center of the loop in the absence of the shell is  $H = \frac{4\pi}{c} \left(\frac{1}{2a}\right)$  one obtains for the case of a loop concentric with the shell:

$$\frac{H(R=0) \text{ with shell}}{H(R=0) \text{ without shell}} = \alpha = \left(\frac{R_1}{R_2}\right) \frac{1}{\left[1 + \frac{1}{3}(\theta a)^2 R_2 d\right]}$$

This is precisely the result for the uniform applied field  $\left(\frac{R_1}{R_2} \approx 1\right)$ . The radius of the loop, "a", does not appear in  $\alpha$ , roughly speaking, because the dimensions of the source are larger than those of the shell.

Now suppose that the loop is contracted toward some point over the shell, or alternatively the shell is expanded until the loop faces essentially a plane sheet of superconductor. Let the sheet be perfectly plane ( $R_2 = \infty$ ). This problem has been treated in detail elsewhere.<sup>3</sup> In cylindrical coordinates with the center of coordinates at the center of the loop and the z-axis along the axis of the loop, the axial magnetic field behind the film at

$[-(z_0 - d) > z > -z_0]$  is to a sufficient approximation:

$$H_z(r=0, z < -z_0) = \frac{4\pi}{c} \frac{1}{(\varnothing\varnothing)^2 d} \left[ \frac{3 x a^2}{r a^2 + x^2} \right]^{5/2}$$

where  $x \equiv |z| - d + 2(\log 2) \left( \frac{\cosh \varnothing^2 d}{\varnothing^2} \right)$  and "a" is the radius of the loop.

In the absence of the shell, this field is:

$$H_{z0} = \frac{2\pi}{c} \frac{a^2}{[a^2 + z^2]^{3/2}}$$

and the attenuation factor becomes:

$$\frac{H_z}{H_{z0}} = \alpha = \frac{6}{(\varnothing\varnothing)^2 d} \left\{ \frac{x[a^2 + z^2]^{3/2}}{[a^2 + x^2]^{5/2}} \right\}$$

Since the term

$$\left[ 2(\log 2) \left\{ \frac{\cosh(\varnothing^2 d)}{\varnothing^2} \right\} - d \right]$$

is of order  $\lambda$ , we write  $x \approx z$  and have:

$$\alpha = \left\{ \frac{1}{(\varnothing\varnothing)^2 d \left[ \frac{a^2 + z^2}{6z} \right]} \right\}$$

which exhibits the explicit dependence of  $\alpha$  on the diameter of the source. The "size parameter" of the spherical shell, R, has been replaced by the function

$$\left[ \frac{a^2 + z^2}{6z} \right]$$

which is  $\geq a/3$ . Hence  $\alpha_{\max} = \frac{3}{[(\varnothing\varnothing)^2 ad]}$ .

Here  $\alpha$  is smaller than the attenuation factor expected from  $\alpha' = [1 + L/m]^{-1}$ .  $L$  introduces, at most, the familiar  $\log(8a/b)$  term of the current loop, where "a" is the mean diameter and "b" is the mean cross section of the induced current distribution  $m$  is proportional to "a". Hence  $L/m \approx \log(a/b)$  and  $\alpha'$  is a conservative estimate of the actual attenuation.

### CONCLUSIONS

The discussion has shown that  $\alpha' = [1 + (L/m)]^{-1}$  is a useful measure of the magnetic field attenuation for thin walled superconducting shells. We have shown that it is not necessary in practice to know the exact value of  $(L/m)$ , which would necessitate solving the attenuation problem in detail for every particular situation, but that it is only necessary to observe the relative size and position of shell and sources in order to be able to determine the value of the length  $x$  in  $[(\pi d)^2 \times d] \gtrsim (L/m)$ , i.e., in order to know the attenuation. From the results it follows that the attenuation is strong for  $x$  of the order of centimeters even if  $d/\lambda \ll 1$ .  $(H_i/H_o) = \alpha$  is of order  $10^{-4}$  for single shells and can easily be of order  $10^{-8}$  for double shells.

Finally we suggest that it may be of considerable interest from a technological point of view to know the shielding strength of such shells. Superconducting magnetometers, for instance, must be carefully screened from stray fields. This discussion shows that a thin evaporated layer of superconducting material can be used very effectively to provide magnetic shielding of circuit components from external fields as well as from one another.



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